

Evolutionarily Stable Strategies of Mutual Help Between Relatives Having Unequal Fertilities

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The evolutionarily stable strategy of mutual help between relatives having unequal fertilities is studied in a kin selection model, which also takes into account competition between kins and the possibility of reciprocation. It turns out that competition and reciprocation can establish ESSs which are completely different from those expected by Hamilton's basic theory.

1. Introduction

If two relatives each have a different expected fertility, we anticipate the evolutionary stable altruistic behavior between them to imply a different intensity of altruism in each direction. Thus, if the expected fertility of individual *A* is smaller than that of its relative *B*, it is anticipated that the degree of altruism from *A* to *B* should always be higher than the degree of altruism exhibited in the opposite direction (assuming, of course, a similar but a reversed situation). This is actually implied by the original kin selection model of Hamilton (1964), since any amount of help granted by one of the individuals to its relative, and which increases the survival probability of the recipient by a certain degree, will result in a larger expected increase in fitness for the relative with the higher expected fertility. Hence the gain in the inclusive fitness of *A* (via the kinship component) as a result of helping *B*, is larger than the corresponding gain to *B* in a reversed, similar situation, (see also Eshel & Cohen, 1976; Dawkins 1976).

It is interesting to examine the robustness of these predictions when taking into account additional factors, such as competition between the two relatives and the possibility of reciprocation (Eshel & Cohen, 1976). For that purpose this work presents a kin selection model, similar to that of Eshel & Motro (1981), but instead of assuming that both relatives have the same fertility, here the asymmetric situation is considered, namely that one of the relatives is more fertile than the other. Since a full analytical treatment of the model is very complicated, part of the analysis is substituted by a numerical example. Though having a very limited and non-general character, this example points to the possibility that competition and reciprocation can sometimes establish patterns of behaviour which are completely different from those expected by Hamilton's basic model.

It should be pointed out that this paper deals with one-to-one situations (that is, a single donor and a single recipient). The question of how an individual should

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deploy its altruism among several asymmetric relatives is an important but different one, and has been dealt with by Altmann (1979), Weigel (1981), Schulman & Rubinstein (1983) and others.

2. Presentation of the model

Consider two individuals, *A* and *B*, and let r ($0 \leq r \leq 1$) be the degree of relatedness (measured by Wright's coefficient of relationship) between them. During their mutual lifetime, these two individuals experience frequently occurring interactions with each other, in which each has the possibility to augment its relative's survival probability by b , while decreasing its own survival probability by c , and the c/b ratios are variable with some fixed distribution. What each individual is free to choose is a value x , the threshold value of the ratio c/b below which it will act. We shall call this strategy the maximal risk strategy, and refer to its value x as the maximal risk value (MRV) (the maximal risk value which maximizes the inclusive fitness in Hamilton's basic kin selection model is $x = r$).

It is convenient to extend the notion of maximal risk to include also negative values. Thus, the maximal risk value of an individual is $x < 0$ if, whenever this individual can inflict some damage on its relative, it will do so as long as the risk it takes in that act is not larger than the proportion $-x$ of the damage to its relative (damage and risk measured in terms of survival probabilities). The MRV $x = 0$ describes the indifference behavior.

The paper deals with a time-continuous model, where each individual has an infinitesimal probability of dying during any infinitesimal time interval dt . It is assumed that this probability depends both on the willingness of the individual to take risks in order to help (or harm) its relative (i.e. the individual's own MRV) and on the willingness of the relative to take risks in order to help (or harm) our individual (i.e. the relative's MRV), provided both individuals are alive at the beginning of the time interval in question. We shall denote this probability for *A* by $\lambda_A(x, y) dt$, and the corresponding probability for *B* by $\lambda_B(y, x) dt$, where x and y are the MRV of *A* and *B*, respectively. The death probability of *A* in the absence of *B* is $\lambda_A(0, 0) dt = \lambda_A dt$, say, and the death probability of *B* in the absence of *A* is $\lambda_B(0, 0) dt = \lambda_B dt$. From the definition of y as the MRV of *B* it follows that $\lambda_A(x, y)$ is a decreasing function of y . Likewise, $\lambda_B(y, x)$ is a decreasing function of x .

We assume that $\lambda_A(x, y)$ and $\lambda_B(y, x)$ are constant over time. Hence the random event described by the death of either relative constitutes a Poisson process with the parameter $\lambda_A(x, y) + \lambda_B(y, x)$. From that it follows that the length of time during which both individuals are alive is exponentially distributed, with expectation $1/[\lambda_A(x, y) + \lambda_B(y, x)]$. Similarly, the distribution of the remaining life span of each individual after the death of its relative is also exponential, with expectations $1/\lambda_A$ for *A* and $1/\lambda_B$ for *B*.

The number of offspring produced by each individual is assumed to be proportional to the length of time it lives in the presence of its relative, plus $1 + \sigma$ times the length of time it lives after the death of its relative (provided the individual in

question outlives its relative), where $\sigma \geq 0$ is the degree of competition between the two relatives. Hence, since the probability that B is the first to die is $\lambda_B(y, x)/[\lambda_A(x, y) + \lambda_B(y, x)]$, the Fisherian fitness of A is

$$\omega_A(x, y) = \frac{1}{\lambda_A(x, y) + \lambda_B(y, x)} + \frac{1 + \sigma}{\lambda_A} \frac{\lambda_B(y, x)}{\lambda_A(x, y) + \lambda_B(y, x)} \quad (1)$$

and the Fisherian fitness of B is

$$\omega_B(y, x) = \frac{1}{\lambda_A(x, y) + \lambda_B(y, x)} + \frac{1 + \sigma}{\lambda_B} \frac{\lambda_A(x, y)}{\lambda_A(x, y) + \lambda_B(y, x)} \quad (2)$$

(note that the assumption that fertility is proportional to life span implies that possible difference in longevity between the two relatives will create a corresponding difference in their fertilities).

The inclusive fitness of A can be presented (Eshel & Cohen, 1976; Eshel & Motro, 1981) as

$$\Omega_A(x, y) = \omega_A(x, y) + r\omega_B(y, x) \quad (3)$$

and the inclusive fitness of B

$$\Omega_B(y, x) = \omega_B(y, x) + r\omega_A(x, y). \quad (4)$$

Given the MRV of the other player, the inclusive fitness of each relative changes as a function of its own MRV. In the next section it is shown that for any MRV of B, for example, a unique MRV of A exists which maximizes A's inclusive fitness (i.e. which is A's best reply to B).

3. Some Analysis of the Model

First, note that for each x and y the following relations exist:

$$x \frac{\partial}{\partial x} \lambda_B(y, x) = - \frac{\partial}{\partial x} \lambda_A(x, y) \quad (5)$$

$$y \frac{\partial}{\partial y} \lambda_A(x, y) = - \frac{\partial}{\partial y} \lambda_B(y, x). \quad (6)$$

These are actually a restatement of the definition of maximal risk: increasing the MRV of A by dx results in a decrease of the death probability of B during a time interval of length dt (at the beginning of which both are alive) by $\lambda_B(y, x) dt - \lambda_B(y, x + dx) dt$. If $x \geq 0$, the increase in the death probability of A is $\lambda_A(x + dx, y) dt - \lambda_A(x, y) dt$. From the definition of maximal risk, the ratio of the increase in the death probability of A to the decrease in the death probability of B should be between x and $x + dx$. Letting dx tend to zero, one obtains eqn (5). A similar argument shows the validity of eqn (6) also for $x < 0$. (The proof of eqn (6) is identical.)

Since $(\partial/\partial y)\lambda_A(x, y) \leq 0$ and $(\partial/\partial x)\lambda_B(y, x) \leq 0$, it follows, using eqns (5) and (6), that

$$\begin{aligned} \frac{\partial}{\partial x} \lambda_A(x, y) &\leq 0 && \text{for } x \leq 0 \\ &\geq 0 && \text{for } x \geq 0 \end{aligned}$$

and

$$\begin{aligned} \frac{\partial}{\partial y} \lambda_B(y, x) &\leq 0 && \text{for } y \leq 0 \\ &\geq 0 && \text{for } y \geq 0. \end{aligned}$$

These clearly illustrate the fact that by taking no risks in order to help or harm its relative, the individual thus minimizes its own death probability.

We now return to eqn (3) and find that for each MRV y of B, there exists a single MRV $x^* = x^*(y)$ that maximizes the inclusive fitness of A. We shall consider three different assumptions pertaining to the functions $\lambda_A(x, y)$ and $\lambda_B(y, x)$.

Assumption 1: $\lambda_A(\alpha, \beta) < \lambda_B(\alpha, \beta)$ for all α, β (particularly, $r\lambda_A < \lambda_B$).

In this case (see Appendix), $-1 < x^*(y) < 1$ and $x^*(y)$ is an increasing function of y for $-\infty < y \leq 1$.

Assumption 2: $\lambda_A(\alpha, \beta) > \lambda_B(\alpha, \beta)$ for all α, β , but $r\lambda_A < \lambda_B$.

In this case, $-\infty < x^*(y) < 1$ and $x^*(y)$ is an increasing function of y , $-\infty < y \leq 1$ (see Appendix).

Assumption 3: $\lambda_A(\alpha, \beta) > \lambda_B(\alpha, \beta)$ for all α, β , and also $r\lambda_A > \lambda_B$.

Here, $x^*(y)$ can be either finite (and if so, $x^*(y) > 1$), $x^*(y)$ can be $-\infty$ or it can be $+\infty$ (again, see Appendix).

4. The Evolutionarily Stable Strategies of Mutual Help

Though not always the case, here maximization of inclusive fitness is used as a criterion for evolution. Following Maynard Smith (1977) and Grafen & Sibly (1978), (\hat{x}, \hat{y}) will be called an evolutionarily stable strategy (ESS) of our two types of player game if $\Omega_A(x, \hat{y}) < \Omega_A(\hat{x}, \hat{y})$ for all $x \neq \hat{x}$ and $\Omega_B(y, \hat{x}) < \Omega_B(\hat{y}, \hat{x})$ for all $y \neq \hat{y}$. (See also the Discussion in Taylor, 1979.)

The ESSs of our model are the intersection points (\hat{x}, \hat{y}) of the curves $x^*(y)$ and $y^*(x)$ (provided such intersections exist). This is because \hat{x} is A's best reply to \hat{y} , and \hat{y} is B's best reply to \hat{x} . Hence, if the population is fixed at (\hat{x}, \hat{y}) , the inclusive fitness of any A mutant, for example, having $x \neq \hat{x}$ as its MRV, will be smaller than the inclusive fitness of the prevailing A individuals, having the value \hat{x} .

Whereas all intersection points (\hat{x}, \hat{y}) of the best reply curves $x^*(y)$ and $y^*(x)$ are stable according to the evolutionary stability criterion (i.e. a population which is fixed at (\hat{x}, \hat{y}) cannot be invaded neither by A nor by B mutants), there are intersection points which exhibit an additional form of stability. These are the points where, when viewed in the x - y plane, the curve $y^*(x)$ intersects the curve $x^*(y)$ from above. (That is to say, for all $y \neq \hat{y}$ sufficiently close to \hat{y} , the following holds: $x^*(y) < \hat{x}$ if and only if $y < y^*(x^*(y))$.) These ESS (and only these) feature a property similar to the continuous stability defined by Eshel & Motro (1981) (see also Eshel, 1983) for the one type of player game with a continuous strategy set: if the population

maintains any strategy (x, y) close enough to such an ESS, a dynamic selection process will eventually lead to the establishment of that ESS in the population.

To find the evolutionarily stable strategies, we assume that the expected fertility of A is smaller than that of B. More specifically, we assume that $\lambda_A(\alpha, \beta) > \lambda_B(\alpha, \beta)$ for all α, β .

Case 1: $\lambda_A(\alpha, \beta) > \lambda_B(\alpha, \beta)$ for all α, β , but $r\lambda_A < \lambda_B$ (i.e. the expected fertility of A is always smaller than that of B, but in the absence of the other, A's expected fertility is larger than r times B's expected fertility). In this case, since $y^*(x)$ is an increasing function of x ($-\infty < x \leq 1$), with $-1 < y^*(x) < 1$ (see Assumption 1), and since $x^*(y)$ is an increasing function of y ($-\infty < y \leq 1$), with $x^*(y) < 1$ (see Assumption 2), there is at least one intersection point (\hat{x}, \hat{y}) where $y^*(x)$ intersects $x^*(y)$ from above. This ESS satisfies $\hat{x} < 1$ and $-1 < \hat{y} < 1$, and it also features the dynamic stability described above.

A sufficient condition for the existence of ESS of mutual help (i.e. such that both \hat{x} and \hat{y} are positive) is that both $x^*(0) > 0$ and $y^*(0) > 0$, and it turns out (see Appendix) that a necessary and sufficient condition for that is that competition between the two relatives is not too strong, such that for each, the inclusive fitness under the assumption of mutual indifference ($x = y = 0$) is larger than its expected fitness had the other relative ceased to exist. (Because the expected fertility of A is assumed to be smaller than that of B, it can be shown that the validity of the above statement with regard to B implies its validity also with regard to A. Thus, the necessary condition for a positive ESS is, indeed, somewhat simpler.)

If both $r = 0$ and $\sigma = 0$, the mutual indifference behavior $(\hat{x}, \hat{y}) = (0, 0)$ is an ESS.

Case 2: $\lambda_A(\alpha, \beta) > \lambda_B(\alpha, \beta)$ for all α, β , and also $r\lambda_A > \lambda_B$ (i.e. the expected fertility of A is always smaller than that of B. Moreover, in the absence of the other, A's expected fertility is smaller than r times B's expected fertility.) This case is not treated analytically, but is illustrated in the example of Section 5.

5. An Example

This section presents a numerical example which illustrates the ESS of mutual help (or spite) between two relatives (A and B), where the expected fertility of A is smaller than that of B.

Assume

$$\lambda_A(x, y) = \lambda_A - (1+x) \exp(-x) + \exp(-y)$$

and

$$\lambda_B(y, x) = \lambda_B - (1+y) \exp(-y) + \exp(-x),$$

where $\lambda_A > \lambda_B > 1$. Hence $\lambda_A(\alpha, \beta) > \lambda_B(\alpha, \beta)$ for all α, β (i.e. A is less fertile than B).

For different values of $\lambda_A > \lambda_B$, the ESS (\hat{x}, \hat{y}) is calculated as a function of the degree of relatedness r and of the degree of competition σ .

		$\lambda_A = 3$		$\lambda_B = 2$			
		$\sigma = 0$	$\sigma = \frac{1}{4}$	$\sigma = \frac{1}{2}$	$\sigma = \frac{3}{4}$	$\sigma = 1$	$\sigma = 2$
$r = \frac{1}{2}$	\hat{x}	0.776	0.723	0.670	0.619	0.568	0.378
	\hat{y}	0.405	0.319	0.247	0.184	0.129	-0.041
$r = \frac{1}{4}$	\hat{x}	0.408	0.287	0.174	0.071	-0.024	-0.326
	\hat{y}	0.224	0.121	0.032	-0.046	-0.115	-0.326
$r = \frac{1}{8}$	\hat{x}	0.209	0.062	-0.068	-0.183	-0.283	-0.565
	\hat{y}	0.119	0.005	-0.093	-0.177	-0.249	-0.450
$r = 0$	\hat{x}	0.000	-0.163	-0.299	-0.411	-0.503	-0.738
	\hat{y}	0.000	-0.123	-0.225	-0.309	-0.377	-0.545
		$\lambda_A = 5$		$\lambda_B = 2$			
		$\sigma = 0$	$\sigma = \frac{1}{4}$	$\sigma = \frac{1}{2}$	$\sigma = \frac{3}{4}$	$\sigma = 1$	$\sigma = 2$
$r = \frac{1}{2}$	\hat{x}	1.224	1.285	1.348	1.412	1.478	1.757
	\hat{y}	0.274	0.208	0.159	0.120	0.089	0.011
$r = \frac{1}{4}$	\hat{x}	0.642	0.557	0.476	0.398	0.322	0.049
	\hat{y}	0.160	0.089	0.033	-0.013	-0.052	-0.171
$r = \frac{1}{8}$	\hat{x}	0.325	0.184	0.053	-0.066	-0.175	-0.517
	\hat{y}	0.088	0.009	-0.056	-0.111	-0.160	-0.307
$r = 0$	\hat{x}	0.000	-0.179	-0.333	-0.464	-0.576	-0.878
	\hat{y}	0.000	-0.090	-0.164	-0.226	-0.278	-0.414
		$\lambda_A = 10$		$\lambda_B = 2$			
		$\sigma = 0$	$\sigma = \frac{1}{4}$	$\sigma = \frac{1}{2}$	$\sigma = \frac{3}{4}$	$\sigma = 1$	$\sigma = 2$
$r = \frac{1}{2}$	\hat{x}	2.309	2.691	3.097	3.527	3.984	6.138
	\hat{y}	0.160	0.120	0.091	0.069	0.052	0.009
$r = \frac{1}{4}$	\hat{x}	1.237	1.299	1.362	1.426	1.490	1.757
	\hat{y}	0.104	0.067	0.041	0.022	0.008	-0.026
$r = \frac{1}{8}$	\hat{x}	0.632	0.544	0.458	0.375	0.293	-0.010
	\hat{y}	0.063	0.022	-0.008	-0.032	-0.053	-0.115
$r = 0$	\hat{x}	0.000	-0.203	-0.387	-0.550	-0.696	-1.122
	\hat{y}	0.000	-0.054	-0.099	-0.139	-0.175	-0.279

REMARKS

(1) As expected, both \hat{x} and \hat{y} in our example are increasing functions of the degree of relatedness r .

(2) If $r\lambda_A < \lambda_B$, both \hat{x} and \hat{y} are decreasing functions of the degree of competition σ (i.e. the larger the competition between the two relatives, the smaller is their tendency to help each other).

However, if $r\lambda_A > \lambda_B$, then \hat{y} is a decreasing function of σ , but \hat{x} (which is larger than 1) is an *increasing* function of σ . This interesting finding is due to the fact that $r\lambda_A > \lambda_B$ means that in the absence of the other, B contributes *more* to the inclusive

fitness of A (via the kinship component) than A contributes to its own inclusive fitness. Greater competition means more room for A when B is absent, but, more important to A's welfare, it means more room for B when A is absent!

(3) In our example, if $\hat{y} > 0$ then $\hat{x} > \hat{y}$ (i.e. if the ESS of the more fertile type is that of positive altruism, the ESS of the less fertile type is that of even greater altruism). This is the situation anticipated by Hamilton's original model, and it can be shown that this is always the case whenever $\lambda_A(\alpha, \beta) - \lambda_B(\alpha, \beta) = \text{constant} (> 0)$.

(4) If $\hat{y} < 0$, then \hat{x} can either be positive (i.e. the more fertile individual behaves spitefully in return to its relative's altruism!) or \hat{x} can be negative (this negative value can be either larger or smaller than $\hat{y} < 0$).

6. Conclusions

Taking only relatedness into account, Hamilton's kin selection model predicts that the degree of altruism from a less fertile individual towards its more fertile relative should be larger than the degree of altruism exhibited in the opposite direction.

However, when also considering competition between kins and the possibility of reciprocation, we see that the mutual evolutionarily stable behavior is not that simple. Situations can exist, for example, in which the ESS is that of altruistic behavior in one direction (from the less fertile towards the more fertile relative), but spiteful behavior in the opposite direction. More intense competition can lead to situations where the ESS is that of mutual spitefulness, with different degrees of spite in each direction—in some cases the ESS is such that the more fertile individual is the more hostile, whereas in other cases the less fertile relative is the one which exhibits the greater amount of spitefulness.

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APPENDIX

Finding the Best Reply in Each of the Three Assumptions of Section 3

Differentiating eqn (3) with respect to x , and applying eqn (5), we get

$$\frac{\partial}{\partial x} \Omega_A(x, y) = \frac{-(\partial/\partial x)\lambda_B(y, x)}{[\lambda_A(x, y) + \lambda_B(y, x)]^2} \times \left\{ (1+r)(1-x) - (1+\sigma) \left(\frac{1}{\lambda_A} - \frac{r}{\lambda_B} \right) [\lambda_A(x, y) + x\lambda_B(y, x)] \right\}.$$

Let

$$\pi = (1+\sigma)(1/\lambda_A - r/\lambda_B), \quad (\text{A1})$$

$$\phi_A(x; y) = (1+r)(1-x) - \pi[\lambda_A(x, y) + x\lambda_B(y, x)]. \quad (\text{A2})$$

Since

$$\frac{-(\partial/\partial x)\lambda_B(y, x)}{[\lambda_A(x, y) + \lambda_B(y, x)]^2} \geq 0$$

for all x and y , the sign of $\phi_A(x; y)$ is identical to the sign of $(\partial/\partial x)\Omega_A(x, y)$.

We have

$$\phi_A(-1; y) = 2(1+r) - \pi[\lambda_A(-1, y) - \lambda_B(y, -1)] \quad (\text{A3})$$

$$\phi_A(1; y) = -\pi[\lambda_A(1, y) + \lambda_B(y, 1)] \quad (\text{A4})$$

$$\frac{d}{dx} \phi_A(x; y) = -(1+r) - \pi\lambda_B(y, x) \quad (\text{A5})$$

$$\frac{d^2}{dx^2} \phi_A(x; y) = -\pi \frac{\partial}{\partial x} \lambda_B(y, x). \quad (\text{A6})$$

Assumption 1: $\lambda_A(\alpha, \beta) < \lambda_B(\alpha, \beta)$ for all α, β (particularly, $r\lambda_A < \lambda_B$, hence $\pi > 0$).

From eqns (A4) and (A5) we immediately see that $\phi_A(1; y) < 0$ and $(d/dx)\phi_A(x; y) < 0$ (i.e. $\phi_A(x; y)$ is a decreasing function of x). Using eqn (6) we get

$$\frac{d}{dy} [\lambda_A(-1, y) - \lambda_B(y, -1)] = (1+y) \frac{d}{dy} \lambda_A(-1, y).$$

Hence $\lambda_A(-1, y) - \lambda_B(y, -1)$ has a global maximum at $y = -1$, and from eqn (A3)

$$\phi_A(-1; y) \geq 2(1+r) - \pi[\lambda_A(-1, -1) - \lambda_B(-1, -1)] > 0.$$

Thus $\phi_A(x; y)$ intersects the x axis at a single point x^* , $-1 < x^* < 1$. Since the intersection is from above, $\Omega_A(x, y)$ obtains there a global maximum in x . In other words, for each MRV y of B there is a single value $x^*(y)$ ($-1 < x^* < 1$) which maximizes the inclusive fitness of A.

By differentiating $\phi_A(x^*; y) = 0$ with respect to y , we get

$$\frac{d}{dy} x^*(y) = \frac{-\pi(1-x^*y)[(\partial/\partial y)\lambda_A(x, y)|_{x=x^*}]}{1+r+\pi\lambda_B(y, x^*)}$$

which is positive for $x^*y < 1$. Since $x^* < 1$, $x^*(y)$ is an increasing function of y for $-\infty < y \leq 1$.

A necessary and sufficient condition for $x^*(y) > 0$ is $\phi_A(0; y) > 0$, i.e. $1+r-\pi\lambda_A(0, y) > 0$. In particular, $x^*(0) > 0$ if and only if $1+r-(1+\sigma)(1-r\lambda_A/\lambda_B) > 0$. This condition is satisfied if and only if competition between the two relatives is not too strong, such that the inclusive fitness of A under the assumption of mutual indifference behavior ($x = y = 0$) is larger than its expected fitness had B ceased to exist.

Assumption 2: $\lambda_A(\alpha, \beta) > \lambda_B(\alpha, \beta)$ for all α, β , but $r\lambda_A < \lambda_B$ (hence $\pi > 0$).

From eqns (A4), (A5) and (A6) we see that $\phi_A(x; y)$ is a decreasing and convex function of x (for all y), with $\phi_A(1; y) < 0$. Hence $\phi_A(x; y)$ intersects the x axis at a single point x^* , $-\infty < x^* < 1$. Since the intersection is from above, $\Omega_A(x, y)$ obtains there a global maximum in x . Using the same argument as in Assumption 1, we get that $x^*(y)$ is an increasing function of y , $-\infty < y \leq 1$. The necessary and sufficient condition for $x^* > 0$ is the same as in Assumption 1.

Assumption 3: $\lambda_A(\alpha, \beta) > \lambda_B(\alpha, \beta)$ for all α, β , and also $r\lambda_A > \lambda_B$ (hence $\pi < 0$).

From eqns (A4) and (A6) we see that $\phi_A(x; y)$ is a concave function of x , with $\phi_A(1; y) > 0$. The situation is either one of the three possibilities:

(i) $\phi_A(x; y)$ always increases in x , thus it intersects the x axis at a single point $-\infty < x_1 < 1$. At that point the intersection is from below, and $\Omega_A(x, y)$ obtains there its global minimum in x . $\Omega_A(x, y)$ obtains its global maximum at either $x = -\infty$ or $x = +\infty$. Hence the best reply $x^*(y)$ is either $-\infty$ or $+\infty$.

(ii) $\phi_A(x; y)$ always decreases in x , thus it intersects the x axis at a single point $1 < x_2 < \infty$. At that point the intersection is from above, and $\Omega_A(x, y)$ obtains there its global maximum in x . Hence $x^*(y) = x_2(y)$ ($x_2(y) > 1$).

(iii) $\phi_A(x; y)$ intersects the x axis twice, at x_1 and at x_2 (where $-\infty < x_1 < 1 < x_2 < \infty$). At x_1 the intersection is from below, and $\Omega_A(x, y)$ obtains its global minimum in x at that point. At x_2 the intersection is from above, thus $\Omega_A(x, y)$ obtains there a local maximum. Hence $x^*(y)$ is either $-\infty$ or $x_2(y)$ ($x_2(y) > 1$).